QCD in the nuclear medium
and effects due to Cherenkov gluons

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Abstract

The equations of in-medium gluodynamics are proposed. Their classical lowest order solution is explicitly shown for a color charge moving with constant speed. For nuclear permittivity larger than 1 it describes emission of Cherenkov gluons resembling results of classical electrodynamics. The values of the real and imaginary parts of the nuclear permittivity are obtained from the fits to experimental data on the double-humped structure around the away-side jet obtained at RHIC. The dispersion of the nuclear permittivity is predicted by comparing the RHIC, SPS and cosmic ray data. This is important for LHC experiments. Cherenkov gluons may be responsible for the asymmetry of dilepton mass spectra near ρ-meson observed in the SPS experiment with excess in the low-mass wing of the resonance. This feature is predicted to be common for all resonances. The "color rainbow" quantum effect might appear according to higher order terms of in-medium QCD if the nuclear permittivity depends on color.

1   Introduction

The collective effects observed in ultrarelativistic heavy-ion collisions at SPS and RHIC [1, 2, 3, 4, 5, 6] have supported the conjecture of quark-gluon plasma (QGP) formed in these processes. The properties and evolution of this medium are widely debated. At the simplest level it is assumed to consist of a set of current quarks and gluons. It happens however that their interaction is quite strong so that the notion of the strongly interacting quark-gluon plasma (sQGP) has been introduced. Moreover, this substance reminds an ideal liquid rather than a gas. Whether perturbative quantum chromodynamics (pQCD) is applicable to the description of the excitation modes of this matter is doubtful. Correspondingly, the popular theoretical approaches use either classical solutions of in-vacuum QCD equations at the initial stage or hydrodynamics at the final stage of its evolution. However, it is surprising that no attempts to write down the equations of in-medium QCD, similar to

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the very successful approach in electrodynamics, were done before the recent paper [7] appeared.

The collective excitation modes of the medium may play a crucial role. One of the ways to gain more knowledge about the excitation modes is to consider the propagation of relativistic partons through this matter. Phenomenologically their impact would be described by the nuclear permittivity of the matter corresponding to its response to passing partons. Namely this approach is most successful for electrodynamical processes in matter. Therefore it is reasonable to modify QCD equations by taking into account collective properties of the quark-gluon medium. For the sake of simplicity we consider here the gluodynamics only. The generalization to quarks is straightforward.

The classical lowest order solution of these equations coincides with Abelian electrodynamical results up to a trivial color factor. One of the most spectacular of them is Cherenkov radiation and its properties. Now, Cherenkov gluons take place of Cherenkov photons [8, 9]. Their emission in high energy hadronic collisions is described by the same formulae but with nuclear permittivity in place of the usual one. Actually, one considers them as quasiparticles, i.e. quanta of the medium excitations with properties determined by the permittivity.

These formulae are used for fits of experimental data on the double-humped structure around the away-side jet obtained at RHIC. The values of the real and imaginary parts of the nuclear permittivity are determined. Comparing the RHIC, SPS and cosmic ray data one can guess that the nuclear permittivity depends on energy. This leads to predictions for future LHC experiments.

Beside the high energy region, the effects due to Cherenkov gluons might be noticed in asymmetry of the shapes of resonances passing through the nuclear medium. This prediction is compared with experimental data of SPS.

Another possible effect of "color rainbow" might appear if the nuclear permittivity differs for partons of different colors. The higher-order non-linear terms of in-medium QCD equations are in charge of it.

2 Equations of in-medium gluodynamics

At the beginning let us remind the classical in-vacuum Yang-Mills equations

$$D_\mu F^{\mu\nu} = J^\nu, \tag{1}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu], \tag{2}$$

where $A^\mu = A^\mu_a T_a$; $A_a(A^0_a \equiv \Phi_a, \mathbf{A}_a)$ are the gauge field (scalar and vector) potentials, the color matrices T_a satisfy the relation $[T_a, T_b] = if_{abc}T_c$, $D_\mu = \partial_\mu - ig[A_\mu, \cdot]$, $J^\nu(\rho, \mathbf{j})$ is a classical source current, $\hbar = c = 1$ and the metric tensor is $g^{\mu\nu}=\text{diag}(+,-,-,-)$.

In the covariant gauge $\partial_\mu A^\mu = 0$ they are written as

$$\Box A^\mu = J^\mu + ig[A_\nu, \partial^\nu A^\mu + F^{\nu\mu}], \tag{3}$$

where $\Box$ is the d'Alembertian operator. It was shown [10] (and is confirmed in what follows) that in this gauge the classical gluon field is given by the solution of the corresponding Abelian problem.

The chromoelectric and chromomagnetic fields are

$$E^\mu = F^{\mu 0}, \tag{4}$$

$$B^\mu = -\frac{1}{2}\epsilon^{\mu i j} F^{ij}, \tag{5}$$

or as functions of gauge potentials in vector notations

$$\mathbf{E}_a = -\text{grad}\Phi_a - \frac{\partial \mathbf{A}_a}{\partial t} + g f_{abc}\mathbf{A}_b\Phi_c, \tag{6}$$

$$\mathbf{B}_a = \text{curl}\mathbf{A}_a - \frac{1}{2}g f_{abc}[\mathbf{A}_b\mathbf{A}_c]. \tag{7}$$

The equations of motion (1) in vector form are written as

$$\text{div}\mathbf{E}_a - g f_{abc}\mathbf{A}_b\mathbf{E}_c = \rho_a, \tag{8}$$

$$\text{curl}\mathbf{B}_a - \frac{\partial \mathbf{E}_a}{\partial t} - g f_{abc}(\Phi_b\mathbf{E}_c + [\mathbf{A}_b\mathbf{B}_c]) = \mathbf{j}_a. \tag{9}$$

The Abelian equations of in-vacuum electrodynamics are obtained from Eq. (3) if the second term in its right-hand side is put equal to zero and color indices omitted. The medium is accounted if $\mathbf{E}$ is replaced by $\mathbf{D} = \epsilon\mathbf{E}$ in $F^{\mu\nu}$, i.e. in Eq. $(4)^2$. Therefore the Eqs. (8), (9) in vector form are most suitable for their generalization to in-medium case. The equations of in-medium electrodynamics differ from in-vacuum ones by dielectric permittivity $\epsilon \neq 1$ entering there as

$$\triangle \mathbf{A} - \epsilon\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mathbf{j}, \tag{10}$$

$^2\epsilon$ denotes the dielectric permittivity of the medium. It is well known [11] that magnetic properties of a substance are reproduced with the proper account of temporal and spatial dispersion of ϵ.

$$\epsilon(\triangle\Phi - \epsilon\frac{\partial^2\Phi}{\partial t^2}) = -\rho. \tag{11}$$

The permittivity describes the matter response to the induced fields which is assumed to be linear and constant in Eqs. (10), (11). It is determined by the distribution of internal current sources in the medium. Then external currents are only left in the right-hand sides of these equations.

Now, the Lorentz gauge condition is

$$\mathrm{div}\mathbf{A} + \epsilon\frac{\partial\Phi}{\partial t} = 0. \tag{12}$$

The Lorentz invariance is lost if $\epsilon \neq 1$ in front of the second terms in the left-hand sides. Then one has to deal within the coordinate system where a substance is at rest. The values of ϵ are determined just there. To cancel these requirements one must use Minkowski relations between $\mathbf{D}$, $\mathbf{E}$, $\mathbf{B}$, $\mathbf{H}$ valid for a moving medium [12]. It leads to more complicated formulae, and we do not use them in this paper.

The most important property of solutions of these equations is that while the in-vacuum ($\epsilon = 1$) equations do not admit any radiation processes, it happens for $\epsilon \neq 1$ that there are solutions of these equations with non-zero Poynting vector.

Now we are ready to write down the equations of in-medium gluodynamics generalizing Eq. (3) in the same way as Eqs. (10), (11) are derived in electrodynamics. We introduce the nuclear permittivity and denote it also by ϵ since it will not lead to any confusion. After that one should replace $\mathbf{E}_a$ in Eqs. (8), (9) by $\epsilon\mathbf{E}_a$ and get:

$$\epsilon(\mathrm{div}\mathbf{E}_a - gf_{abc}\mathbf{A}_b\mathbf{E}_c) = \rho_a, \tag{13}$$

$$\mathrm{curl}\mathbf{B}_a - \epsilon\frac{\partial\mathbf{E}_a}{\partial t} - gf_{abc}(\epsilon\Phi_b\mathbf{E}_c + [\mathbf{A}_b\mathbf{B}_c]) = \mathbf{j}_a. \tag{14}$$

The space-time dispersion of ϵ is neglected here.

In terms of potentials these equations are cast in the form:

$$\triangle\mathbf{A}_a - \epsilon\frac{\partial^2\mathbf{A}_a}{\partial t^2} = -\mathbf{j}_a - gf_{abc}(\frac{1}{2}(\mathrm{curl}[\mathbf{A}_b, \mathbf{A}_c] + [\mathbf{A}_b\mathrm{curl}\mathbf{A}_c]) + \frac{\partial}{\partial t}(\mathbf{A}_b\Phi_c) -$$
$$\epsilon\Phi_b\frac{\partial\mathbf{A}_c}{\partial t} - \epsilon\Phi_b\mathrm{grad}\Phi_c - \frac{1}{2}gf_{cmn}[\mathbf{A}_b[\mathbf{A}_m\mathbf{A}_n]] + g\epsilon f_{cmn}\Phi_b\mathbf{A}_m\Phi_n), \tag{15}$$

$$\triangle\Phi_a - \epsilon\frac{\partial^2\Phi_a}{\partial t^2} = -\frac{\rho_a}{\epsilon} + gf_{abc}(2\mathbf{A}_b\mathrm{grad}\Phi_c + \mathbf{A}_b\frac{\partial\mathbf{A}_c}{\partial t} - \epsilon\frac{\partial\Phi_b}{\partial t}\Phi_c) +$$
$$g^2 f_{amn}f_{nlb}\mathbf{A}_m\mathbf{A}_l\Phi_b. \tag{16}$$

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If the terms with explicitly shown coupling constant g are omitted, one gets the set of Abelian equations which differ from electrodynamical equations (10), (11) by the color index a only. Their solutions are shown in the next section. The external current is ascribed to a parton fast moving relative to other partons "at rest". The crucial distinction between Eq. (3) and Eqs. (15), (16) is that there is no radiation (the field strength is zero in the forward light-cone and no gluons are produced) in the lowest order solution of Eq. (3) and it is admitted for Eqs. (15), (16) because ϵ takes into account the collective response (polarization) of the nuclear matter.

The omitted above terms are of the order of g^3 because the potentials and the classical current J^μ are linear in g. They can be taken into account as a perturbation. It was done in [13, 14] for in-vacuum gluodynamics. For in-medium gluodynamics they are considered in Section 7 and Ref. [15].

3 Cherenkov gluons as the classical lowest order solution of in-medium gluodynamics

The classical solution of Eqs. (15), (16) immediately leads to the notion of Cherenkov gluons at $\epsilon > 1$ in analogy with Cherenkov photons in electrodynamics. The unique feature is independence of the coherence of subsequent emissions by an external current on the time interval between these processes.

The problem of the coherence length for Cherenkov radiation was extensively studied [16, 17]. It was shown that the ω-component of the field of a current can be imitated by a set of oscillators with frequency ω situated along the trajectory. The waves from all oscillators add up in the direction given by the Cherenkov angle θ independent on the length of the interval filled in by these oscillators. The phase disbalance $\Delta\phi$ between emissions with frequency $\omega = k/\sqrt{\epsilon}$ separated by the time interval Δt (or the length $\Delta z = v\Delta t$) is given by

$$\Delta\phi = \omega\Delta t - k\Delta z\cos\theta = k\Delta z(\frac{1}{v\sqrt{\epsilon}} - \cos\theta) \qquad (17)$$

up to terms which vanish for large distances between oscillating sources and the detector. For Cherenkov effects the angle θ is

$$\cos\theta = \frac{1}{v\sqrt{\epsilon}}. \qquad (18)$$

The coherence condition $\Delta\phi = 0$ is valid independent of Δz. This is a crucial prop-

erty specific for Cherenkov radiation only[3]. Thus the change of color at emission vertices is not important if one considers a particular a-th component of color fields produced at Cherenkov angle. Therefore the fields $(\Phi_a, \mathbf{A}_a)$ and the classical current for in-medium gluodynamics can be represented by the product of their electrodynamical expressions $(\Phi, \mathbf{A})$ and the color matrix T_a. As a result, one can neglect the "rotation" of color at emission vertices and use in the lowest order for Cherenkov gluons the well known formulae for Cherenkov photons just replacing α by $\alpha_S C_A$ for gluon currents in probabilities of their emission. Surely, there is radiation at angles different from the Cherenkov angle (18). For such gluons one should take into account the coherence length and color rotation considering corresponding Wilson lines [18].

Let us remind the explicit Abelian solution for the current with velocity $\mathbf{v}$ along z-axis

$$\mathbf{j}(\mathbf{r}, t) = \mathbf{v}\rho(\mathbf{r}, t) = 4\pi g \mathbf{v}\delta(\mathbf{r} - \mathbf{v}t). \tag{19}$$

In the lowest order the solutions for scalar and vector potentials are related so that

$$\mathbf{A}^{(1)}(\mathbf{r}, t) = \epsilon \mathbf{v}\Phi^{(1)}(\mathbf{r}, t), \tag{20}$$

where the superscript (1) indicates the solutions of order g.

Therefore the explicit expressions for Φ suffice. Using the Fourier transform, the lowest order solution of Eq. (11) with account of (19) can be cast in the form

$$\Phi^{(1)}(\mathbf{r}, t) = \frac{g}{2\pi^2 \epsilon} \int d^3 k \frac{\exp[i\mathbf{k}(\mathbf{r} - \mathbf{v}t)]}{k^2 - \epsilon(\mathbf{k}\mathbf{v})^2}. \tag{21}$$

The integration over the angle in cylindrical coordinates gives the Bessel function $J_0(k_\perp r_\perp)$. Integrating over the longitudinal component k_z with account of the poles due to the denominator[4] and then over the transverse one $k_\perp$, one gets the following expression for the scalar potential [19]

$$\Phi^{(1)}(\mathbf{r}, t) = \frac{2g}{\epsilon} \frac{\theta(vt - z - r_\perp \sqrt{\epsilon v^2 - 1})}{\sqrt{(vt - z)^2 - r_\perp^2(\epsilon v^2 - 1)}}. \tag{22}$$

Here $r_\perp = \sqrt{x^2 + y^2}$ is the cylindrical coordinate, z is the symmetry axis. The cone

$$z = vt - r_\perp \sqrt{\epsilon v^2 - 1} \tag{23}$$

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[3]The requirement for $\Delta\phi$ to be a multiple of 2π (or a weaker condition of being less or of the order of 1) in cases when Cherenkov condition is not satisfied imposes limits on the effective radiation length as it happens, e.g., for Landau-Pomeranchuk or Ter-Mikaelyan effects.

[4]These poles are at work only for Cherenkov radiation!

determines the position of the shock wave due to the θ-function in Eq. (22). The field is localized within this cone. The Descartes components of the Poynting vector are related according to Eqs. (22), (20) by the formulae

$$S_x = -S_z \frac{(z-vt)x}{r_\perp^2}, \quad S_y = -S_z \frac{(z-vt)y}{r_\perp^2}, \tag{24}$$

so that the direction of emitted gluons is perpendicular to the cone (23) and defined by the Cherenkov angle

$$\tan^2 \theta = \frac{S_x^2 + S_y^2}{S_z^2} = \epsilon v^2 - 1, \tag{25}$$

which coincides with (18).

The higher order terms (g^3 ...) can be calculated using Eqs. (15), (16) (see Section 7).

The expression for the intensity of the radiation is given by the Tamm-Frank formula (up to Casimir operators)

$$\frac{dW}{dz} = 4\pi\alpha_S \int \omega d\omega (1 - \frac{1}{v^2\epsilon}). \tag{26}$$

It is well known that it leads to infinity for constant ϵ. The ω-dependence of ϵ (its dispersion) usually solves the problem.

For absorbing media ϵ acquires the imaginary part $\epsilon = \epsilon_1 + i\epsilon_2$. The sharp front edge of the shock wave is smoothed. The angular distribution of Cherenkov radiation widens. The δ-function at the angle (18), (25) is replaced by the a'la Breit-Wigner angular shape [20] with maximum at the angle given by (18), (25) but with $|\epsilon|$ in place of ϵ and the width proportional to the imaginary part.

The energy loss dW per the length dz is determined by the formula

$$\frac{dW}{dz} = -gE_z. \tag{27}$$

In the lowest order

$$E_z^{(1)} = i \int \frac{d^4k}{(2\pi)^4} [\omega A_z^{(1)}(\mathbf{k}, \omega) - k_z \Phi^{(1)}(\mathbf{k}, \omega)] e^{i(\mathbf{kv}-\omega)t}, \tag{28}$$

and in k-representation one gets in QCD

$$\Phi_a^{(1)} = 2\pi g Q_a \frac{\delta(\omega - kv\zeta)v^2\zeta^2}{\omega^2\epsilon(\epsilon v^2\zeta^2 - 1)}, \tag{29}$$

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$$A_{z,a}^{(1)} = \epsilon v \Phi_a^{(1)}, \tag{30}$$

$$\zeta = \cos\theta. \tag{31}$$

The subscript of ϵ_a is omitted because there will be no summation on it even at higher order (see Section 7).

Thus, for ultrarelativistic radiating centers, the differential Cherenkov energy loss spectrum in the opaque medium per length dz reads [20, 21, 22]

$$\frac{1}{\omega}\frac{dW}{dz\,d\omega\,d\phi\,d\cos\theta} = \frac{4\alpha_S C_V}{\pi}\frac{\cos\theta(1-\cos^2\theta)\Gamma(\omega)}{(\cos^2\theta - \zeta(\omega))^2 + \Gamma^2(\omega)}, \tag{32}$$

where θ, ϕ are polar and azimuthal angles with respect to the emitter propagation direction,

$$\begin{aligned}
\zeta(\omega) &= \frac{\epsilon_1(\omega)}{\epsilon_1^2(\omega) + \epsilon_2^2(\omega)}, \\
\Gamma(\omega) &= \frac{\epsilon_2(\omega)}{\epsilon_1^2(\omega) + \epsilon_2^2(\omega)},
\end{aligned} \tag{33}$$

so that $\zeta(\omega)$ controls the location of the maximum and $\Gamma(\omega)$ controls the spreading around it. For $(\epsilon_2/\epsilon_1)^2 \ll 1$, the energy-dependent maximum of the differential spectrum (32) is at

$$\cos^2\theta_{\max}(\omega) \approx \frac{\epsilon_1(\omega)}{\epsilon_1^2(\omega) + \epsilon_2^2(\omega)}. \tag{34}$$

Without absorption, the potential (22) is infinite on the cone. With absorption, it is finite everywhere except the cone vertex and is inverse proportional to the distance from the vertex. Absorption induces also longitudinal excitations (chromoplasmons) which are proportional to the imaginary part of ϵ and usually small compared to transverse excitations.

Several experimental observations may be explained as stemming from emission of Cherenkov gluons. Most accurate are the RHIC data with high statistics about the two-humped structure of particle distributions around the away-side jets in central nucleus-nucleus collisions at $\sqrt{s} = 200$ GeV per nucleon [1, 2, 3, 4, 5, 6]. Their fit allowed to get values of the real and imaginary parts of the nuclear permittivity [22]. The earlier cosmic ray event [23], which initiated the idea about Cherenkov gluons, showed the smaller values of the nuclear permittivity [24, 25]. Quite small values of ϵ were also obtained from low-statistics samples of nuclei interactions at

SPS energies [26, 27, 28]. This indicates on energy dependence (dispersion) of ϵ. The SPS data [29] on asymmetry of the shapes of resonances passing through the nuclear medium can also be interpreted in terms of Cherenkov gluons [30]. Let us start with fits of the RHIC data.

4   Comparison with RHIC data

In Eq. (32) the angle θ of emission of Cherenkov gluons is measured with respect to the direction of propagation of the radiating color current. In the experimental setup used at RHIC the special trigger method has been used. Among all central Au-Au collisions at 200 GeV per nucleon there were chosen only those where the partons of colliding nuclei scattered at the angle about $\pi/2$. One of them passed a very thin layer of a nucleus and produced the jet similar to those in pp-collisions. Another one traversed through the whole nucleus and formed the away-side jet. Namely around this jet the rings of hadrons were detected and interpreted as originating from Cherenkov gluons. The partons initiating the hard away-side jets play the role of the radiating color currents. Since the trigger is placed at $\pi/2$ to the colision axis of initial partons, the trigger parton is detected at this angle. The away-side parton is also at this angle in the opposite direction if the energies of colliding partons are equal. It was shown in [25] that the mismatch of these energies is unimportant because the structure functions decrease fast for larger mismatch. The background from gluons radiated by the aside moving partons is low and can result in a slight widening of observed humps. Due to the forward-backward symmetry it does not influence the positions of the maxima. Thus, in the first approximation, for analytical estimates one can consider only those partons whose direction of propagation is orthogonal to the collision axis z.

In order to compare the spectrum (32) with the experimental data one should rewrite it in terms of experimental laboratory polar and azimuthal angles θ_L and ϕ_L. It is easy to see that for the above geometry

$$\cos\theta \;=\; |\sin\theta_L \cos\phi_L|,$$

for ϕ_L counted from the away-side jet.

The number dN of emitted gluons per length dz with energy and angles within intervals $d\omega\, d\theta_L\, d\phi_L$ is

$$\frac{dN}{dz\, d\omega\, d\phi_L\, d\cos\theta_L} = \frac{4\alpha_S C_V}{\pi} \frac{|\sin\theta_L \cos\phi_L|(1 - \sin^2\theta_L \cos^2\phi_L)\Gamma(\omega)}{\left(\sin^2\theta_L \cos^2\phi_L - \zeta(\omega)\right)^2 + \Gamma^2(\omega)}. \tag{35}$$

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The azimuthal gluon distribution $dN/d\phi_L$ is obtained from Eq. (35) by integrating over θ_L. This can be done analytically:

$$
\frac{dN}{d\omega\, dl\, d\phi_L} = 4\alpha_s C_V \frac{\Gamma}{|\cos\phi_L|} \left\{ \frac{1}{\sqrt{\Gamma^2 + (\cos^2\phi_L - \zeta)^2}} \frac{1}{\sqrt{A^2 + B^2}} \times \right.
$$

$$
\left[\left(A + (1-\zeta)\frac{B}{\Gamma} \right) \sqrt{\frac{1}{2}\left(\sqrt{A^2 + B^2} + A \right)} - \right.
$$

$$
\left. \left. \left(B + (1-\zeta)\frac{A}{\Gamma} \right) \sqrt{\frac{1}{2}\left(\sqrt{A^2 + B^2} - A \right)} \right] - 1 \right\}, \tag{36}
$$

where

$$
A(\omega, \phi_L) = \Gamma^2(\omega) + \zeta^2(\omega) - \zeta(\omega)\cos^2\phi_L, \quad B(\omega, \phi_L) = \Gamma(\omega)\cos^2\phi_L. \tag{37}
$$

The gluon spectrum (36) reveals the double-humped structure. The original flow of Cherenkov glue described by Eq. (36) should, of course, transform into that of final hadrons. This can be done only with the help of Monte-Carlo models.

To describe properly the angular pattern observed in the flow of final hadrons in high energy nuclear collisions [1, 6, 3, 4] in terms of the Cherenkov gluon radiation one has to

(a) Describe a kinematical pattern characterizing the initially produced hard partons serving as colored sources of gluon Cherenkov radiation taking into account the experimental cuts on pseudorapidity and transverse momentum [6, 4].

(b) Write down a spectrum of Cherenkov gluon radiation in the opaque medium for the above-described color sources.

(c) Describe a conversion of Cherenkov gluons into observed hadrons taking into account the experimental cuts on transverse momenta of final hadrons [6, 4].

These three steps are realized in a Monte-Carlo procedure. In total the model includes three parameters, described in more details below - two related to the Cherenkov gluon radiation, the magnitudes of the real ϵ_1 and imaginary ϵ_2 parts of the permittivity of the medium, and one related to rescattering of Cherenkov gluons in the process of hadronization $\Delta_\perp$. The values of these parameters are fitted to achieve the best possible agreement with the experimental spectra [6, 4].

The description of the initial hard color sources was performed through a Monte Carlo simulation of hard parton-parton scattering at RHIC energies with PYTHIA [31]. This authomatically allowed to take into account possible mismatch of energies

of initial colliding partons. An initial pool of two-parton configurations resulting from these scatterings consisted of those in which the trigger jet gave rise to a pion hitting the trigger intervals in rapidity, $|\eta|^{\rm tr}_{\rm star} \leq 0.7$ and $|\eta|^{\rm tr}_{\rm phenix} \leq 0.35$, and transverse momentum, $3\,{\rm GeV} \leq |\mathbf{p}^{\rm lab}_{\perp}| \leq 4\,{\rm GeV}$ for STAR and $2\,{\rm GeV} \leq |\mathbf{p}^{\rm lab}_{\perp}| \leq 3\,{\rm GeV}$ for PHENIX [6, 4]. The fragmentation of gluon (quark) generating the trigger near-side jet into pions was described by standard fragmentation functions [32].

The away-side jet gave rise to a pion hitting the rapidity intervals $|\eta|^{\rm aw}_{\rm star} \leq 1$ and $|\eta|^{\rm aw}_{\rm phenix} \leq 0.35$, and transverse momentum, $1\,{\rm GeV} \leq |\mathbf{p}^{\rm lab}_{\perp}| \leq 2\,{\rm GeV}$ for STAR and $2\,{\rm GeV} \leq |\mathbf{p}^{\rm lab}_{\perp}| \leq 3\,{\rm GeV}$ for PHENIX [6, 4]. For simplicity we have considered only the dominating subset of events with gluonic away-side jet.

The angular distribution of emitted Cherenkov gluons is given by the expresion (36). Let us stress once again that in our Monte-Carlo procedure we consider a more general situation where the direction of the gluon generating the away-side jet is fixed within each configuration satisfying the above-described trigger conditions imposed on the properties of the near-side jet in experiment.

To realize a Monte-Carlo procedure of generating the Cherenkov spectrum we have to specify the functions $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$. Taking into account that the experimental cuts on (laboratory frame) intervals of transverse momenta of the away-side hadrons are very strict, $\delta\mathbf{p}^{\rm lab}_{\perp}| \leq 1\,{\rm GeV}$ both for STAR [6] and PHENIX [4] we can with a good accuracy neglect the effects of dispersion and consider energy-independent $\epsilon_{1,2} = {\rm const}$. The values of these constants are determined through fitting the experimental data. Within this assumption the spectrum of produced Cherenkov glue is simply energy-independent, $dN/d\omega\, dl = {\rm const}$. Realistically the possibility of Cherenkov emission is restricted to some finite interval of energies $\omega \leq \omega_{\rm max}$ so that

$$\frac{dN}{d\omega\, dl} \propto \theta\left(\omega_{\rm max} - \omega\right) , \qquad (38)$$

where $\omega_{\rm max}$ is the highest characteristic resonance excitation energy of the medium[5]. In our simulations we have chosen $\omega_{\rm max} = 3.5\,{\rm GeV}$ in accordance with the upper limits of experimental intervals. We have verified that our results are in fact weakly sensitive to the exact value of $\omega_{\rm max}$.

The thus generated flow of Cherenkov glue should, of course, be transformed into that of final hadrons. There exist several phenomenological schemes describing this conversion. In our case it is convenient to use a language of fragmentation functions $D^h_g(x, \mathbf{p}_{\perp}|\,Q^2)$ which generically describe a probability for a gluon with energy E and

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[5]For a discussion of interrelation between Cherenkov gluons and hadronic resonances see [30] and section 6.

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virtuality Q^2 to produce a hadron with the energy xE and transverse momentum $\mathbf{p}_\perp$ measured with respect to the direction of propagation of the initial gluon.

One is first tempted to ascribe the same shape to the spectrum of Cherenkov hadrons relying on the soft blanching hypothesis, equivalent to assuming $D_g^h(x, \mathbf{p}_\perp | Q^2) \propto \delta(1 - x)$ supported by the experimental evidence obtained in the physics of multiparticle production from $e^+e^- \rightarrow$ jets. However, this does not lead to the fully satisfactory description of experimental data because of predicting, in contradiction with the experimental data, that the probability of radiating Cherenkov glue at angles $|\phi_L| \geq \pi/2$ with respect to the away-side jet is strictly zero. These "dead zones" appear because the Cherenkov gluon "halo" can not spread to angles larger than $\pi/2$ to the direction of the emitter. It is clear that such "dead zones" will in fact be present for any fragmentation mechanism that does not generate transverse momentum with respect to the direction of propagation of the initial Cherenkov gluon which in our case corresponds to considering a $\mathbf{p}_\perp$-independent fragmentation function $D_g^h(x, | Q^2)$. We confine our consideration to fragmentation to light hadrons because experimentally the share of other bosonic resonances is quite small (in particular, $\rho : \omega : \phi$=10:1:2 according to [29]). In our analysis we used a simple parametrization of the fragmentation function $D_g^h(x, \mathbf{p}_\perp | Q^2)$:

$$
D_g^h(x, \mathbf{p}_\perp | Q^2) \propto (1 - x)^3 \frac{1}{\sqrt{2\pi\Delta_\perp^2}} \exp\left\{ -\frac{\mathbf{p}_\perp^2}{2\Delta_\perp^2} \right\} , \tag{39}
$$

where the x-dependence is that of a gluon fragmentation function at the reference scale $Q_0 = 2\,\text{GeV}$ [32]. Phenomenologically, the parameter $\Delta_\perp$ accounts for transverse momenta acquired by pions both due to the decay of intermediate resonances and because of rescattering processes.

Thus, we have a set of three parameters: ϵ_1, ϵ_2 and a fraction $\Delta_\perp$.

The values of the parameters that provide the best description of experimental data of STAR and PHENIX collaborations are shown in Table 1.

Table 1

Experiment	$\theta_{\max}$	ϵ_1	ϵ_2	$\Delta_\perp$
STAR	1.04 rad	5.4	0.7	0.7 GeV
PHENIX	1.27 rad	9.0	2.0	1.1 GeV

Let us stress that a difference in the fitted values of ϵ_1 for STAR and PHENIX originates from different positions of angular maxima $\theta_{\max}$ in the corresponding

experimental data. (May be, it could indicate on necessity to take the dispersion into account.) However these values are quite stable in both above approaches because they are defined by maxima positions but not by the humps widths.

The value of ϵ_2, on the contrary, is influenced by the widths. Nevertheless, the ratio $(\epsilon_2/\epsilon_1)^2 \leq 0.05 \ll 1$ stays quite small in all cases.

The transverse smearing parametrized by $\Delta_\perp$ looks very reasonable at the hadronic scale.

The resulting angular spectra for STAR and PHENIX are shown in Figs. 1 and 2 correspondingly. We see that the positions of the maxima (and therefore the values of ϵ_1) are quite stable to accounting for additional smearing on top of that in Eq. (36). However, it is indeed important in achieving a good description of the widths of humps in experimental data as seen in Figs. 1 and 2. The shape of humps in the former "dead zone" determines the parameter $\Delta_\perp$, which, in its turn, influences ϵ_2.

To conclude, the values of the real part of the nuclear permittivity ϵ_1 found from the fit to experimental data of RHIC (albeit somewhat different in the two experimental sets) are determined with good precision. Their common feature is that they are noticeably larger than 1. This shows that the density of scattering centers is quite large and the nuclear medium reminds a fluid rather than a gas (for more details see [25]). The accuracy of the estimate of the values of ϵ_2 is much less but more important is the fact that they are rather small compared with ϵ_1.

5 Cosmic ray events and SPS data

The RHIC experiments with high statistics used a special geometry of events and corresponding triggers to detect rings of hadrons around the away-side jets. Lower statistics data obtained at SPS and in cosmic rays also gave some indications on ring-like events. The very first events with the ring-like structure were observed in non-trigger cosmic ray experiments. Namely they initiated the idea about Cherenkov gluons [8]. Two rings more densely populated by particles than their surroundings were noticed in the cosmic ray event [23] initiated by a primary with energy about 10^{16} eV close to LHC energies. It is demonstrated in Fig. 3 where the number of produced particles is plotted as a function of the logarithm of the distance from the collision axis (proportional to pseudorapidity at small angles). It clearly shows two maxima. This event has been registered in the detector with nuclear and X-ray emulsions during the balloon flight at the altitude about 30 km. Approximately at the same time the similar event with two peaks was observed at 10^{13} eV [34]. Two

colliding nuclei must give rise to two peaks. Some events with one peak (due to the limited acceptance of the installation?) were shown even earlier [35, 36]. The peaks were interpreted as effects due to Cherenkov gluons emitted by the forward and backward moving initial high energy partons. When the two-dimensional distribution of particles was considered in the azimuthal plane (called as target diagram in cosmic rays experiments), this event revealed two (forward and backward in c.m.s.) densely populated ring-like regions within two narrow intervals of polar angles (corresponding to peaks in Fig. 3) but widely distributed in azimuthal angles for each of them. Therefore such events were called as ring-like events.

It was shown that the ring-like structure can be revealed by the event-by-event wavelet analysis [37, 38].

The argument in favor of high energy Cherenkov gluons stems from the behavior of the real parts of hadronic forward scattering amplitudes $\mathrm{Re}F(E, 0^o)$. The intensity of the Cherenkov effect is proportional to the excess of the permittivity ϵ or of the refractive index n over 1. They are related by the formula $\epsilon = n^2$. The general relation of the scattering theory (see, e.g., [39]) between the refractive index and the forward scattering amplitude $F(E, 0^o)$ states that

$$\Delta n = \mathrm{Re}n - 1 = \frac{2\pi N \mathrm{Re}F(E, 0^o)}{E^2} = \frac{3m_\pi^3 \nu_h}{2E^2}\mathrm{Re}F(E) = \frac{3m_\pi^3 \nu_h}{8\pi E}\sigma(E)\rho(E). \quad (40)$$

Here E denotes the energy, N is the density of the scatterers (inhomogeneities) of the medium, ν_h is the number of scatterers within a single nucleon, m_π the pion mass, $\sigma(E)$ the cross section and $\rho(E)$ the ratio of real to imaginary parts of the forward scattering amplitude $F(E)$. Thus the emission of Cherenkov gluons is possible only for processes with positive $\mathrm{Re}F(E)$ or $\rho(E)$. As follows from (40), the values of Δn can be completely different at low and high energies because of different values of N and $\mathrm{Re}F(E, 0^o)$ involved. Unfortunately, we are unable to calculate directly in QCD $\mathrm{Re}F(E, 0^o)$ for gluons and have to rely on analogies and our knowledge of properties of hadrons. The only experimental facts we get about this medium are brought by particles registered at the final stage. They have some features in common which (one can hope!) are also relevant for gluons as the carriers of the strong forces. These are the resonant behavior of hadronic amplitudes at rather low energies and positiveness of $\mathrm{Re}F(E, 0^o)$ for all measured hadronic processes at very high energies. Thus we wait for Cherenkov effects in the resonance region and at high energies.

The real parts of the resonance amplitudes are positive in their low-mass wing. This fact will be extensively used in the next section. Here, it is important that the energies of particles produced within the humps in RHIC data are rather low on

the scale of initial energies. Even the energies of jets-parents are lower than 5 GeV. Therefore the whole effect at RHIC can be related to the low-energy region.

In cosmic ray events the energies of partons are high and the effect can be ascribed to the high-energy behavior of $\mathrm{Re}F(E, 0^o)$.

One of the most intriguing problems is to understand properly the fact that the RHIC and cosmic ray data were fitted with very different values of the refractive index close to 3 and 1, correspondingly. This could be interpreted as due to the difference in values of x (the parton share of energy) and Q^2 (the transverse momenta). It is well known that the region of large x and Q^2 corresponds to the dilute partonic system. At low x and Q^2 the density of partons is much higher.

The density of scattering centers at RHIC conditions is very high as estimated from Eq. (40) [25], about tens partons per proton volume. Here one deals with rather low x and Q^2. Therefore, one should expect the large density of partons in this region and high $\Delta n \sim 1$. It is interesting to note that the two-hump structure disappears in RHIC data at higher p_t where the parton density must get lower. It corresponds to smaller n and θ, i.e. humps merge in the main away-side peak.

In the cosmic ray event one observes effect due to leading partons with large x. Also, the experimentalists pointed out that the transverse momenta in this event are quite large. In this region one would expect for low parton density and small $\Delta n \ll 1$.

The ring-like events with values of ϵ close to but larger than 1 were also found from the SPS non-trigger experiments [26, 27, 28].

Thus the same medium can be probably seen as a liquid or a gas depending on the parton energy and transferred momenta. This statement can be experimentally verified by using triggers positioned at different angles to the collision axis and considering different transverse momenta. In that way, the hadronic Cherenkov effect can be used as a tool to scan $(1/x, Q^2)$-plane and plot on it the parton densities corresponding to its different regions. We discuss this problem also in Section 8.

6 Asymmetry of shapes of resonances produced inside the nuclear medium

The low-energy Cherenkov gluons may be at the origin [30] of another interesting effect - the asymmetry of shapes of resonances created inside the nuclear medium. There exist numerous experimental data [40, 41, 29, 42, 43, 44, 45, 46, 47] about the in-medium modification of widths and positions of prominent vector-meson resonances. They are mostly obtained from the shapes of dilepton mass and transverse

momentum spectra in nucleus-nucleus collisions. Such in-medium effects were tied theoretically to chiral symmetry restoration a long time ago [48].

A significant excess of low-mass dilepton pairs yield for ρ meson over expectations from hadronic decays is observed in the high-statistics experiment [29].

Several approaches have been advocated for explanation of the excess. Strong dependence of the parameters of the effective Lagrangian on the temperature and the chemical potential was assumed in [49, 50]. The hydrodynamical evolution was incorporated in [51] to describe the spectra. The QCD sum rules and dispersion relations have been used [52, 53] to show that condensates decrease in the medium leads to both broadening and slight downward mass shift of resonances. The similar conclusions have been obtained from more traditional attempts using either the empirical scattering amplitudes with parton-hadron duality [54, 55] or the hadronic many-body theory [56, 57, 58].

In the latest approach, which pretends to provide the best description of experimental plots, the in-medium V-meson spectral functions are evaluated. The excess of dilepton pairs below ρ-mass is ascribed to antibaryonic effects. This conclusion is the alternative to more common ideas about the chiral restoration at high energies. It asks for some empirical constraints to fit the observed excess.

We proposed [30] another possible source of low-mass lepton pairs. Namely, the emission of Cherenkov gluons may provide a substantial contribution to the low mass region for any resonance.

Qualitatively, the observed low mass excess of lepton pairs is easy to ascribe to the gluonic Cherenkov effect if one reminds that the index of refraction of any medium exceeds 1 within the lower wing of any resonance (the ρ-meson, in particular).

This feature is well known in electrodynamics (see, e.g., Fig. 31-5 in [59]) where the atoms behaving as oscillators emit as Breit-Wigner resonances when get excited. This results in the indices of refraction larger than 1 within their low-energy wings. In QCD, one can imagine that the nuclear index of refraction for gluons in the hadronic medium behaves similarly in the resonance regions. In classical electrodynamics, it is the dipole excitation of atoms in the medium by light which results in the Breit-Wigner shape of the amplitude $F(E, 0^o)$. In hadronic medium, there should be some modes (quarks, gluons or their preconfined bound states, condensates, blobs of hot matter...?) which can get excited by the impinging parton, radiate coherently if $n > 1$ and hadronize at the final stage as hadronic resonances [60, 61].

The scenario, we have in mind, is as follows. Any parton, belonging to a colliding nucleus, can emit a gluon. On its way through the nuclear medium the parton excites some internal modes. Therefore it affects the medium as an "effective" wave

which accounts also for the waves emitted by other scattering centers (see, e.g., [39]). Beside incoherent scattering, there are processes which can be described as the refraction of the initial wave along the path of the coherent wave. The Cherenkov effect is the induced *coherent* radiation by a set of scattering centers placed on the way of propagation of a gluon. That is why the forward scattering amplitude plays such a crucial role in formation of the index of refraction. At low energies its excess over 1 is related to the resonance peaks as dictated by the Breit-Wigner shapes of the amplitudes (for the similar well known explanation in electrodynamics see, e.g., [59]). In experiment, usual resonances are formed during parton interactions and subsequent color neutralization process. Up to now there is no quantitative explanation of resonant amplitudes at the level of QCD-partons. One can only admit some attractive forces acting within definite energy intervals which lead to resonant behavior. In addition to these forces between the individual partons the collective effects can appear in the nuclear medium. As discussed above, there are several indications on such effects in RHIC experiments like J/Ψ-suppression, azimuthal asymmetry v_2, jet quenching, Cherenkov rings etc (see, e.g., [1, 2, 4, 6]). Among them, Cherenkov gluons are distinguished as the result of the fully coherent reaction of the medium. Only in this case the phase of the amplitude does not depend on the time interval between emissions (see Section 4). The necessary condition $n > 1$ is satisfied within the left-wing resonance region and coherent effect at low energies can be observed only there. Similarly to other partons, Cherenkov gluons participate in formation of resonances contributing to their low-mass wing (and, in particular, to dilepton spectra at these energies). Therefore these radiative effects can add to the ordinary ones (strings?) namely in this specific region. This contribution should be proportional to Δn according to the theory [16]. The shock wave formed by Cherenkov gluons [7] pushes out the hadronic states just with masses within the low-mass wings of resonances. Let us note that the common string description of the processes does not take into account collective effects.

Thus, the ordinary Breit-Wigner shape of the cross section for resonance production must be modified by the coherent medium response. The easiest way to observe this effect would be by measuring the dilepton mass spectra for resonances. For all of them, they would acquire the additional term proportional to Δn at masses below the resonance peak. According to Eq. (40) it must be proportional to the real part of the Breit-Wigner amplitude with the ratio of real to imaginary parts of Breit-Wigner amplitudes equal to

$$\frac{\mathrm{Re}F(M, 0^o)}{\mathrm{Im}F(M, 0^o)} = \frac{m_\rho^2 - M^2}{M\Gamma}\theta(m_\rho^2 - M^2). \tag{41}$$

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Here M is the total c.m.s. energy of two colliding objects (the dilepton mass), m_ρ=775 MeV is the in-vacuum ρ-meson mass. Thus its M-dependence is well defined. This term vanishes for $M > m_\rho$ because only positive Δn lead to the Cherenkov effect. Namely it describes the distribution of masses of Cherenkov states.

Therefore, the shape of the mass distribution near the ρ-meson can be described by the following formula[6]

$$\frac{dN_{ll}}{dM} = \frac{A}{(m_\rho^2 - M^2)^2 + M^2\Gamma^2} \left(1 + w\frac{m_\rho^2 - M^2}{M^2}\theta(m_\rho - M) \right) \qquad (42)$$

The first term corresponds to the ordinary Breit-Wigner cross section with a constant normalization factor A. According to the optical theorem it is proportional to the imaginary part of the forward scattering amplitude. The second term is due to the coherent response of the medium and its weight relative to ordinary processes[7] is described by the only adjustable parameter w. As for any classical effect, one can not calculate the cross section for Cherenkov effect and determine the parameter w theoretically. Therefore, we leave w as an adjustable parameter. The only hope is that it is not very small because such an effect has been observed recently in high energy heavy-ion collisions at RHIC (section 4) as the nimbus around away-side jets with rather high probability. Our fit below supports the assumption that the value of w is quite noticeable and corresponds to higher energy observations.

In these formulas, one should take into account the in-medium modification of the height of the peak and its width. In principle, one could consider m_ρ as a free in-medium parameter as well. We rely on experimental findings that its shift in the medium is small. All this asks for some dynamics to be known. In our approach, it is not determined. Therefore, first of all, we just fit the parameters A and Γ by describing the shape of the mass spectrum at $0.75 < M < 0.9$ GeV measured in [29] and shown in Fig. 4. In this way we avoid any strong influence of the ϕ-meson. Let us note that w is not used in this procedure. The values A=104 GeV^3 and $\Gamma = 0.354$ GeV were obtained. The width of the in-medium peak is larger than the in-vacuum ρ-meson width equal to 150 MeV. It agrees with previous findings of widening of in-medium ρ-mesons.

Thus the low mass spectrum at $M < m_\rho$ depends only on a single parameter w which is determined by the (theoretically unknown) relative role of Cherenkov effects and ordinary mechanism of resonance production. It is clearly seen from Eq.

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[6]We consider only ρ-mesons here. To include other mesons, one should evaluate the similar expressions.

[7]We assume it to be independent of M because the relative probability of Cherenkov emission to ordinary processes can hardly change within this rather short interval of masses.

(42) that the role of the second term in the brackets increases for smaller masses M. The excess spectrum in the mass region from 0.4 GeV to 0.75 GeV has been fitted by $w = 0.19$. The slight downward shift about 40 MeV of the peak of the distribution compared with m_ρ may be estimated from Eq. (42) at these values of the parameters. This agrees with the above statement about small shift compared to m_ρ. The total mass spectrum (the dashed line) and its widened Breit-Wigner component (the solid line) according to Eq. (42) with the chosen parameters are shown in Fig. 4. The overall description of experimental points seems quite satisfactory. The contribution of Cherenkov gluons (the excess of the dashed line over the solid one) constitutes the noticeable part at low masses. The formula (42) must be valid in the vicinity of the resonance peak. Thus we use it for masses larger than 0.4 GeV only.

From general principles one would expect slightly lower p_T for low-mass dilepton pairs from coherent Cherenkov processes than for incoherent scattering. Qualitatively, this conclusion is supported by experiment [29].

Whether the in-medium Cherenkov gluonic effect is as strong as shown in Fig. 4 can be verified by measuring the angular distribution of the lepton pairs with different masses. The trigger-jet experiments similar to that at RHIC are necessary to check this prediction. One should measure the angles between the companion jet axis and the total momentum of the lepton pair. The Cherenkov pairs with masses between 0.4 GeV and 0.7 GeV should tend to fill in the rings around the jet axis. The angular radius θ of the ring is determined by the usual condition.

Another way to demonstrate it is to measure the average mass of lepton pairs as a function of their polar emission angle (pseudorapidity) with the companion jet direction chosen as z-axis. Some excess of low-mass pairs may be observed within the rings. Baryon-antibaryon effects can not possess signatures similar to these ones.

Some indications on the substructure with maxima at definite angles have been found at the same energies by CERES collaboration [62]. It is not clear yet if it can be ascribed to Cherenkov gluons. To recover a definite maximum, it would be better to detect a single parton jet, i.e. to have a trigger.

The prediction of asymmetrical in-medium widening of **any** resonance at its low-mass side due to Cherenkov gluons is universal. This universality is definitely supported by experiment. Very clear signals of the excess on the low-mass sides of ρ, ω and ϕ mesons have been seen in [43, 44]. This effect for ω-meson is also studied in [46]. Slight asymmetry of ϕ-meson near 0.9 - 1 GeV is noticeable in the Fig. 4 shown above but the error bars are large there. We did not try to fit it just to deal with as small number of parameters as possible. There are some indications at RHIC (see Fig. 6 in [45]) on this effect for J/ψ-meson. However, the accuracy of RHIC data both for ρ and J/ψ is not enough to get quantitative conclusions.

To conclude, the new mechanism is proposed for explanation of the low-mass excess of dilepton pairs observed in experiment. It is the Cherenkov gluon radiation which adds to the ordinary processes at the left wing of any resonance. Only one adjustable parameter w is used to describe its contribution to the dilepton spectra. The universal nature of their asymmetry for **all** resonances is predicted.

7 Higher order effects leading to the "color rainbow"

It is interesting to note that the general first order solutions of Eqs. (15), (16), explicitly shown by Eqs. (22), (20) for Cherenkov gluons, stay valid at all orders if the nuclear permittivity does not depend on color. The higher order terms contribute only if the permittivity differs for different colors. The general procedure of their calculation is the same as for in-vacuum QCD [10, 13, 14]. After getting explicit lowest order solution one exploits it together with the non-Abelian current conservation condition to find the current component proportional to g^3. Then with the help of Eqs. (15), (16) one finds the potentials up to the order g^3. They can be represented as integrals convoluting the current with the corresponding in-medium Green function. The even higher order corrections may be obtained in the same way. We outline briefly the path to the next order effects and qualitative results.

The third order terms of the potentials can be found [15] from the following expressions

$$\epsilon(\triangle\Phi_a^{(3)} - \epsilon\frac{\partial^2\Phi_a^{(3)}}{\partial t^2}) = -gf_{abc}[\epsilon_f^2(\epsilon_f v^2 - 1) + (2\epsilon + \epsilon_f^2 v^2)\Delta_{bc}]\frac{\partial\Phi_b^{(1)}}{\partial t}\Phi_c^{(1)}, \qquad (43)$$

$$\Delta_{bc} = (\epsilon_b - \epsilon_c)/2; \qquad \epsilon_f = (\epsilon_b + \epsilon_c)/2,$$

$$\triangle A_{z,a}^{(3)} - \epsilon\frac{\partial^2 A_{z,a}^{(3)}}{\partial t^2} = -gf_{abc}[\epsilon_f(\epsilon_f v - 1/v) + \Delta_{bc}(2v(\epsilon + \epsilon_f) - 1/v)]\frac{\partial\Phi_b^{(1)}}{\partial t}\Phi_c^{(1)}. \quad (44)$$

The density of the energy loss is proportional according to Eq. (27) to

$$E_{z,a}^{(3)} = i\int\frac{d^4k}{(2\pi)^4}\left([\omega A_{z,a}^{(3)}(\mathbf{k},\omega) - k\zeta\Phi_a^{(3)}(\mathbf{k},\omega)] - igf_{abc}\int\frac{d^4k'}{(2\pi)^4}A_{z,b}^{(1)}(k')\Phi_c^{(1)}(k - k')\right). \tag{45}$$

The last term corresponds to the truly non-abelian correction. Only the linear terms in Δ_{bc} must be left everywhere because of the antisymmetry of f_{abc}. It demonstrates that the classical first order solution stays valid for the color independent

permittivity. Δ_{bc} can differ from 0 either due to dispersion effects or because of explicit dependence of permittivity on color.

The calculation of the spectra proceeds [15] in the same way as it was done above for first order terms. For the sake of brevity, we reproduce here only the first term in the limit $\epsilon = \epsilon_f$, $v = 1$:

$$\frac{dN_{a(A\Phi)lim}^{(4)}}{dldxd\omega} = \frac{3\alpha_S^2}{4\sqrt{x}} f_{abc}Q_bQ_c\text{Im}\left[\frac{(\epsilon-1)^{1/2}\Delta_{bc}}{\epsilon^2(\epsilon x-1)^{3/2}}\right]. \tag{46}$$

The terms $\propto \frac{\nu\sqrt{x}}{[(x-x_0)^2+(\nu x_0)^2]^{5/4}}$ and $\propto \frac{\nu}{\sqrt{x}[(x-x_0)^2+(\nu x_0)^2]^{3/4}}$ appear which are different (albeit somewhat similar) to the lowest order terms. Now, the shape is different from simple a'la Breit-Wigner form. The typical proportionality to Δn is seen.

To conclude, one can say that in the case of $\Delta_{bc} \neq 0$ the non-abelian color quantum rainbow appears due to the higher order terms.

8 The nuclear permittivity and the rest system of the nuclear matter

In electrodynamics the permittivity of real substances depends on ω. Moreover it has the imaginary part determining the absorption. E.g., $\text{Re}\,\epsilon$ for water (see [63]) is approximately constant in the visible light region ($\sqrt{\epsilon} \approx 1.34$), increases at low ω and becomes smaller than 1 at high energies tending to 1 asymptotically. The absorption ($\text{Im}\,\epsilon$) is very small for visible light but dramatically increases in nearby regions both at low and high frequencies. Theoretically this behavior is ascribed to various collective excitations in the water relevant to its response to radiation with different frequencies. Among them the resonance excitations are quite prominent (see, e.g., [59]). Moreover, the medium considered is at rest and the permittivity values are determined just in this frame. Even in electrodynamics, the quantitative theory of their energy dependence is still lacking, however.

Then, what can we say about the nuclear permittivity and the frame to define it in?

The partons constituting high energy hadrons or nuclei interact during the collision for a very short time. Nevertheless, there are experimental indications that an intermediate state of matter (CGC, QGP, nuclear fluid ...) is formed and evolves. Those are J/ψ-suppression, jet quenching, collective flow (v_2), Cherenkov rings of hadrons etc. They show that there is collective response of the nuclear matter to

color currents moving in it. Unfortunately, our knowledge of its internal excitation modes is very scarce, much smaller than in electrodynamics.

The permittivity is the internal property of a medium which demonstrates the medium response to the *induced* current. Its quantitative description poses problems even in QED. It becomes more difficult task in QCD where confinement is not understood. The attempts to calculate the nuclear permittivity from first principles are not very convincing. It can be obtained from the polarization operator. The corresponding dispersion branches have been computed in the lowest order perturbation theory [64, 65, 66]. Then the properties of collective excitations have been studied in the framework of the thermal field theories (for review see, e.g., [67]). Their results with additional phenomenological ad hoc assumption about the role of resonances were used in a simplified model of scalar fields [9] to show that the nuclear permittivity can be larger than 1 that admits Cherenkov gluons.

Our main goal is to study the medium response to the *external* color current. We prefer to use the general formulae of the scattering theory [39] to estimate the nuclear permittivity. We have to rely on analogies and our knowledge of properties of hadrons. The only experimental facts we get about this medium are brought by particles registered at the final stage. They have some features in common which (one can hope!) are also relevant for gluons as the carriers of the strong forces. Those are the resonant behavior of amplitudes at rather low energies and positive real part of the forward scattering amplitudes at very high energies for hadron-hadron and photon-hadron processes as measured from the interference of the Coulomb and hadronic parts of the amplitudes. $\text{Re}F(0^o, E)$ is always positive (i.e., $n > 1$) within the low-mass wings of the Breit-Wigner resonances. This shows that the necessary condition for Cherenkov effects $n > 1$ is satisfied at least within these two energy intervals. This fact was used to describe experimental observations at SPS, RHIC and cosmic ray energies. The asymmetry of the ρ-meson shape at SPS [29] and azimuthal correlations of in-medium jets at RHIC [6, 5] were explained above by emission of comparatively low-energy Cherenkov gluons [30, 24]. The parton density and intensity of the radiation were estimated. In its turn, cosmic ray data [23] at energies corresponding to LHC ask for very high energy gluons to be emitted by the ultrarelativistic partons moving along the collision axis [8]. Let us note the important difference from electrodynamics where $n < 1$ at high frequencies. For QGP the high-energy condition $n > 1$ is a consequence of its instability.

The in-medium equations are not Lorentz-invariant. There is no problem in macroscopic electrodynamics because the rest system of the macroscopic matter is well defined and its permittivity is considered there. For collisions of two nuclei (or hadrons) it asks for special discussion.

Let us consider a particular parton which radiates in the nuclear matter. It would "feel" the surrounding medium at rest if momenta of all other partons (or constituents of the matter), with which this parton can interact, sum to zero. In RHIC experiments the triggers which registered the jets (created by partons) were positioned at 90° to the collision axis. Such partons should be produced by two initial forward-backward moving partons scattered at 90°. The total momentum of other partons (medium spectators) is balanced because for such geometry the partons from both nuclei play a role of spectators forming the medium. Thus the center of mass system is the proper one to consider the nuclear matter at rest in this experiment. The permittivity must be defined there. The Cherenkov rings consisting of hadrons have been registered around the away-side jet which traversed the nuclear medium. This geometry requires however high statistics because the rare process of scattering of initial partons at 90° has been chosen.

The forward (backward) moving partons are much more numerous and have higher energies. However, one can not treat the radiation of such a primary parton in c.m.s. in the similar way because the momentum of spectators is different from zero i.e. the matter is not at rest. Now the spectators (the medium) are formed from the partons of another (target) nucleus only. Then the rest system of the medium coincides with the rest system of that nucleus and the permittivity should refer to this system. The Cherenkov radiation of such highly energetic partons must be considered there. That is what was done for interpretation of the cosmic ray event in [8]. This discussion clearly shows that one must carefully define the rest system for other geometries of the experiment with triggers positioned at different angles.

Thus our conclusion is that the definition of ϵ depends on the experiment geometry. Its corollary is that partons moving in different directions with different energies can "feel" different states of matter in the **same** collision of two nuclei because of the dispersive dependence of the permittivity. The transversely scattered partons with comparatively low energies can analyze the matter with rather large permittivity corresponding to the resonance region while the forward moving partons with high energies would "observe" low permittivity in the same collision. This peculiar feature can help scan the $(\ln x, Q^2)$-plane as it is discussed in Section 5 and [25]. It explains also the different values of ϵ needed for description of RHIC and cosmic ray data.

These conclusions can be checked at LHC because both RHIC and cosmic ray geometry will become available there. The energy of the forward moving partons would exceed the thresholds above which $n > 1$. Then both types of experiments can be done, i.e. the 90°-trigger and non-trigger forward-backward partons experiments. The predicted results for 90°-trigger geometry are similar to those at RHIC. The

non-trigger Cherenkov gluons should be emitted within the rings at polar angles of tens degrees in c.m.s. at LHC by the forward moving partons (and symmetrically by the backward ones). This idea is supported by events observed in cosmic rays [23, 8, 24]. The experiments with triggers positioned at various angles to the collision axis should be done at the LHC.

9   Conclusions

The equations of in-medium gluodynamics (15), (16) are proposed. They remind the in-medium Maxwell equations with non-Abelian terms added. Their lowest order classical solutions are similar (up to the trivial color factors) to those of electrodynamics (22), especially, for Cherenkov gluons.

Some effects due to Cherenkov gluons at SPS, RHIC, cosmic rays and LHC energies have been discussed. The comparison with experimental data of RHIC allows to determine the real and imaginary parts of the nuclear permittivity. It is related to the forward scattering hadronic amplitudes. This helps interpret the experimental results about the asymmetry of shapes of resonances traversing the nuclear medium. Some indications on the energy dependence (dispersion) of the nuclear permittivity is obtained from comparison of RHIC and cosmic rays data. This asks for the distinction between the different coordinate systems in which the Cherenkov radiation (and nuclear permittivity) should be treated for partons moving in different directions with different energies.

Some estimates of properties of the nuclear matter formed in ultrarelativistic heavy-ion collisions have been done. This consideration predicts new features at the LHC [25].

Acknowledgements

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Figure captions.

Fig. 1. Away-side azimuthal correlations for STAR in the rapidity and transverse momentum intervals indicated in the text and their description by the hypothesis about Cherenkov gluons.

Fig. 2. Away-side azimuthal correlations for PHENIX in the rapidity and transverse momentum intervals indicated in the text and their description by the hypothesis about Cherenkov gluons.

Fig. 3. The distribution of the number of produced particles at different distances from the event axis r in the stratospheric event at 10^{16} eV [23] has two pronounced peaks. Correspondigly, the pseudorapidity distribution possesses two such peaks.

Fig. 4. Excess dilepton mass spectrum in semi-central In(158 AGeV)-In of NA60 (dots) compared to the in-medium ρ-meson peak with additional Cherenkov effect (dashed line).